This article will give a brief mathematical overview of tic-tac-toe introducing ordered pairs, symmetry, probability, and critical thinking. The game of tic-tac-toe is very straightforward. Pick your mark. Get three marks in a row. Any person age four (maybe younger) and above can learn to play. Whoever goes first gets to make more marks (X and O). Is it fair? If this is unfair, is it worth playing for the player who does not make the first mark? Using some mathematical theory, we can analyze the chances of the player who makes his or her mark first (X) versus the other player (O).

(1,3) (2,3) (3,3)

(1,2) (2,2) (3,2)

(1,1) (2,1) (3,1)

Each of the locations that a mark can be placed can be described by an ordered pair (x,y) where $1 \leq x \leq 3$ and $1 \leq y \leq 3$ (x and y are integers). Since there are 3 choices for x and 3 choices for y, there are $3 \times 3 = 9$ choices for the marks. Throughout the discussion, I will use ordered pairs as a way to describe locations of the marks and (1,1) will be the location of the lower left hand corner of the tic-tac-toe grid.

In most games that are played in tic-tac-toe, play begins at the center, the ordered pair (2,2). When analyzing, it can be determined if black (I will refer the first player as “white”, and the second player as “black”) can draw or tie or lose. We will also assume that the player and opponent will always block the opposing player from getting 3 of the same mark in a row.

Again, I am only considering here an analysis based only off of the first two marks made in the game. Throughout the game, there could be various errors, but those errors will not be discussed here since this is an introductory

analysis of tic-tac-toe. This introduction gives an analysis based off of the first two moves of the game. Since there are 9 possibilities for the first mark, there will only be 8 possibilities for the second mark. This gives a total of 9 x 8 or 72 different combinations of the first two moves that will be discussed in this article. Each of these possibilities are numbered in the diagram at the end of the article.

We will begin by the first player (X) choosing the location (2,2) and the second player (O) choosing the location (1,1). No matter where the first player plays the next mark, the second player can draw by blocking the first player at every instance. Because of symmetry, the same scenario would be true if the second player plays in any of the corners (1,3), (3,1), or (3,3). Can you see that this is true by simply rotating the symmetrical tic-tac-toe board?

Suppose the 2nd player plays his first mark at one of the other four squares ((1,2), (2,3), (2,1), (3,2)). The second player loses by force! The first player will play in the corner on move 3 next to the mark made on move 2 (could be either corner). When the block is made by the second player on move 4, the first player (on move 5) plays in the corner which not adjacent to the second player's first mark. Player 1 has two ways to win and the second player can only block one of those ways. This covers games 1-4 and 9-12.

Instead of the usual play in the middle, suppose that the first play is in one of the corners (1,1), (3,1), (1,3), or (3,3) and the second player plays in the middle row(column) non-adjacent to the mark made by the first player. There are 3 ways for the first player to win, 3 ways for a draw, and 1 way for black to win. The first player wins if he makes the next play in the center. Also the first player

wins if the next play is made in the same row (2nd way) or the same column (3rd way). After the second player blocks (move 4) and the first player blocks (move 5), the first player will have two ways to win. After the 2nd player blocks, the 1st player wins by force by playing in the center.

The second player can win if the first player makes the next move (move 3) adjacent to the first and second moves (but not in the center). All other third moves made by the first player will force a draw. This covers diagrams 5 – 8.

If the second player makes the usual play and plays in the center, the first player will force a draw as long as the second player does not make a play so that there are marks completely along the diagonal. If this play is made, then the second player can draw if the fourth move by the second player is not made in the corner. This covers diagrams 13-16.

If the second player makes a play in the center row (column) (not the center) adjacent to the play of the first player (in the corner), the first player wins by force if (1)the third play is also adjacent to the first play (not the center), (2) if the third play is in the center, or (3) the third play is furthest away from the first play, but on the same row (column). If the first player makes the third play along the diagonal, the 2nd player can force a draw.

If the first player makes the third play on the row(column) which makes 3 marks in a row. The 2nd player can only draw if the fourth play is in the center. Otherwise, the first player wins.

If the first player makes the third play adjacent to the 2nd player (not in the corner or center), the second player must play on the diagonal created by the first mark to force a draw, otherwise, the first player wins.

If the first player makes the third-play non-adjacent to both marks, but not in the corners, the 2nd player can force a draw if play is made in the corner.

If white plays in the corner (move 1) and the second player plays in the same row or column and in the corner, the second player can forces a draw if the third play is made by the first player in the center.

The first player wins by force if the second play is in one of the other corners. The first player also wins by force if the third play is made adjacent (not in the same row column as the second play and not in the center) to the first play. Unusually, the second player can actually win if the third play is made by the first player so that 3 marks in the row are made!

If the 1st player makes the third play in the center row (column) non-adjacent to the first two plays (but not in the center), the second player can force a draw only if the 4th play is made in the corner (but not on the long diagonal). All other 4th plays force a win for the first player.

If the third play is made adjacent to the 2nd play (in the center row/column, but not in the center), the 2nd player can only draw if the 4th play is made in the center row or column.

If the first play is made in the center row/column (but not in the center) and the 2nd play is made in the corner not in the same row/column as the first play.

This gives the 2nd player the best probability of winning. A draw will occur if the first player makes the third play in the center.

The first player wins if the third play is made in the middle row/column adjacent to the first play (but not in the center).

If the third play is in the corner non-adjacent to the first mark, then the second player can only force a draw if the 4th play is made in the corner (not on the diagonal of the 2nd play). The first player wins for all other 4th plays.

For all other third plays made by the first player, the second player wins by force! Very Surprising!

Suppose that the 2nd player makes a play in the same row/column (not in the center). A draw is made if the first player plays in either corner non-adjacent to the 2nd play. If the first player's third play is in the center, the 2nd player must play in the corner to force a draw.

If the third play is in the center row.column (not in the center), the 2nd player must play in a corner adjacent to the marks of the first player to force a draw. If the third play is in the corner adjacent to the 2nd play, the second player can draw by playing in one of the corners or adjacent to the third play (not in the center).

If the 2nd player plays in the center, a draw is made if the third play is in the corner adjacent to the first play. If the third play is in the center row(column), the fourth play must be in one of the corners adjacent to the marks made by the first player. If the third play is in the corner non-adjacent to the first mark, a draw

is made as long as the 2nd player does not mark in the center row(column) or the corner non-adjacent to the first mark. If the third mark is made so that ther are 3 marks in a row, the second player wins by playing in the corner!

If the second play is in the corner adjacent to the first play, a draw is made if the third play is in the center. If the third play is adjacent to both marks (first two) (but not in the center), a draw is made if the 4th move is not in the corner.

If the third play is on the long diagonal of the 2nd play in the corner, the 2nd player cannot make the 4th move adjacent to the first play (except for the center), the first player would win if this occurs.

If the third mark is in the corner (non-adjacent to both first and second marks), the second player can only draw if a play is made in the center row (column).If the third mark is made anywhere else, black wins!

If the third play is in the center row/column and adjacent to the first mark made, the 2nd player can draw if the fourth play is in any corner (except the corner non-adjacent to the mark made by the first player).

If the third play is non-adjacent to both marks (first and second), the second player can draw if a play is made in the center on the fourth move.

If the second mark is adjacent to the first mark (but not in the center or corner) and the third mark by the first player is adjacent to both marks, the first player wins.

If the third mark is made in the corner adjacent to the first mark, but not adjacent to the second mark, the second player wins. The 2nd player also wins if

the third mark by the first player is made on the same row/column as the first mark (but not the center).

The last group of moves involve the first two plays on the long diagonal. If the first play makes a mark in the center, it will be a draw. If the third play is adjacent to the first mark (but not in the center), the second player wins.

The first player wins if the third mark is in one of the other corners.

If the third play is in the center row/column non-adjacent to the first play, the second player can draw if the 4th move is made in the center row/column or in the corner non-adjacent to the three marks made.

If the third play in the center row/column and furthest from the first mark, the 2nd player can draw if play is in the corner non-adjacent to all marks, in the center or the location adjacent to the first mark and non-adjacent to the other two marks.

In conclusion, player may choose a tic-tac-toe variation that does not allow a play in the center until the 2nd or 3rd move. This could make the game more interesting and challenging.